

# Modeling change: A gentle introduction to cross-lagged and latent growth curve approach: course materials

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# Cross-lagged models and mediation analysis

*Modeling change: A gentle introduction to cross-lagged and latent  
growth curve approach*

*Mitja Ružojčić*

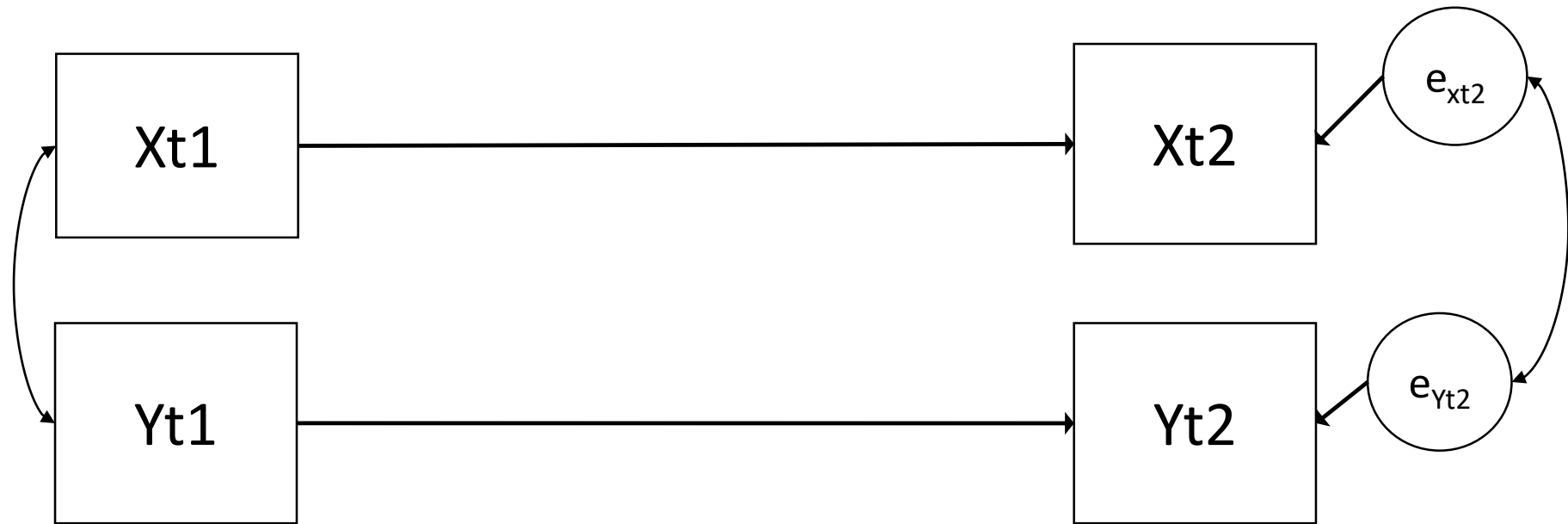
*September 26-27, 2023*

*Zagreb, Croatia*

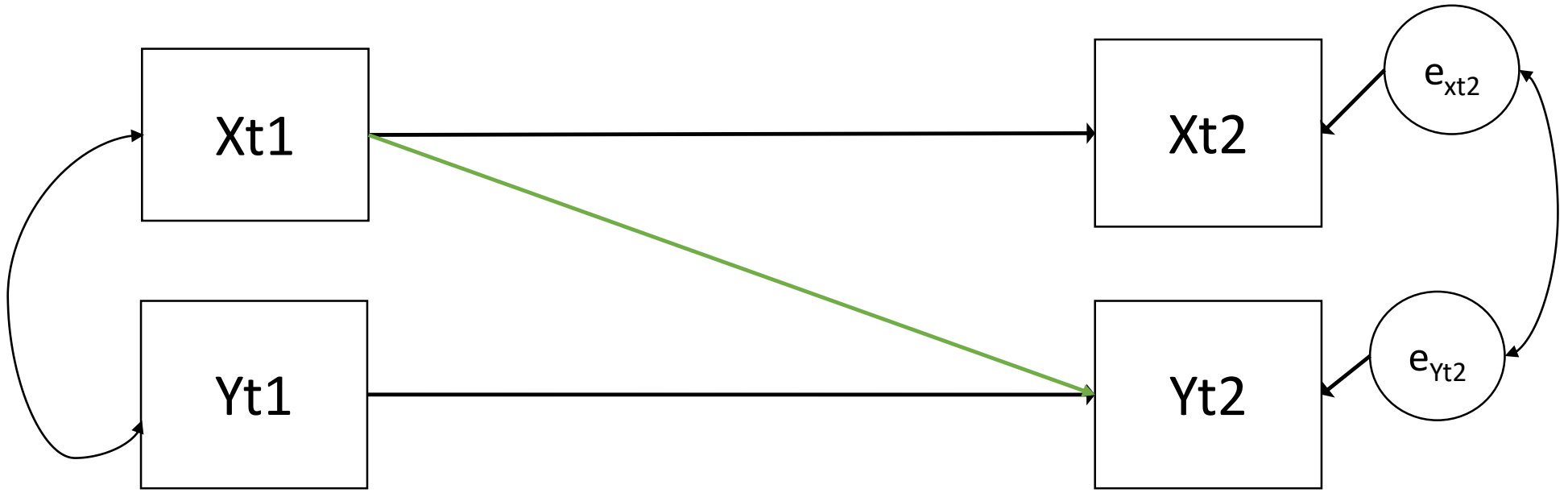
# When do we need cross-lagged?

- Primary reason – we want to investigate causal directionality of the relationships between variables.
- Needed for mediational models where causality is implied.
- Next best thing for determining causality after...
- ...experiments.

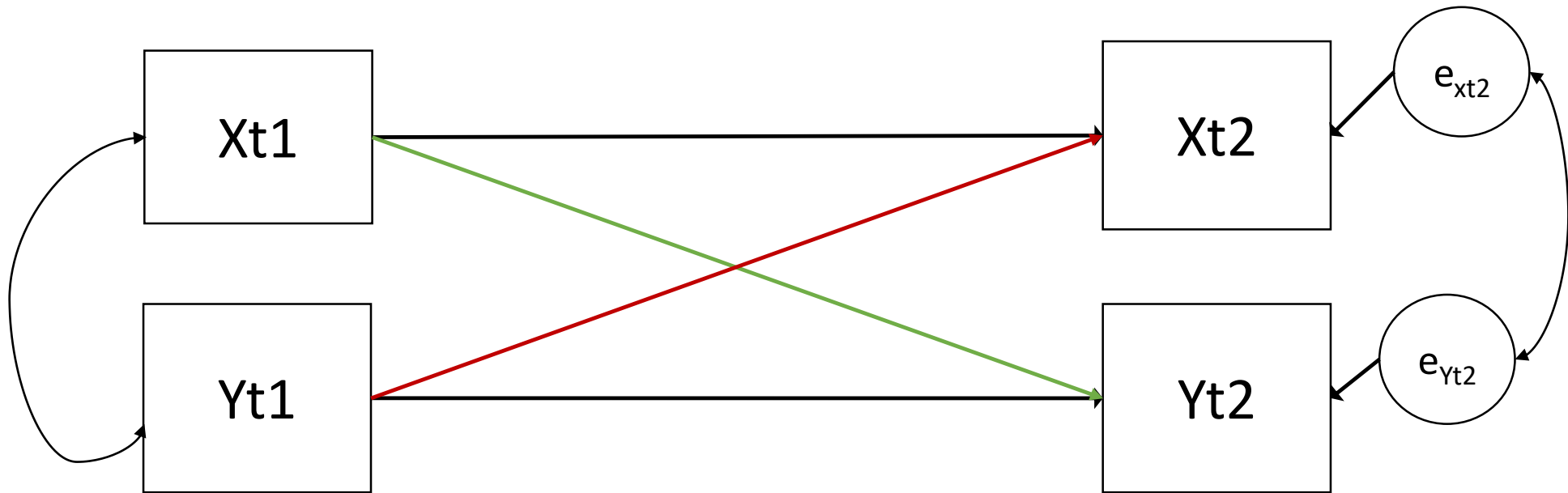
First building block of cross-lagged model is  
**autoregressive model**



# Adding the first cross-lagged effect



# Adding the second cross-lagged effect

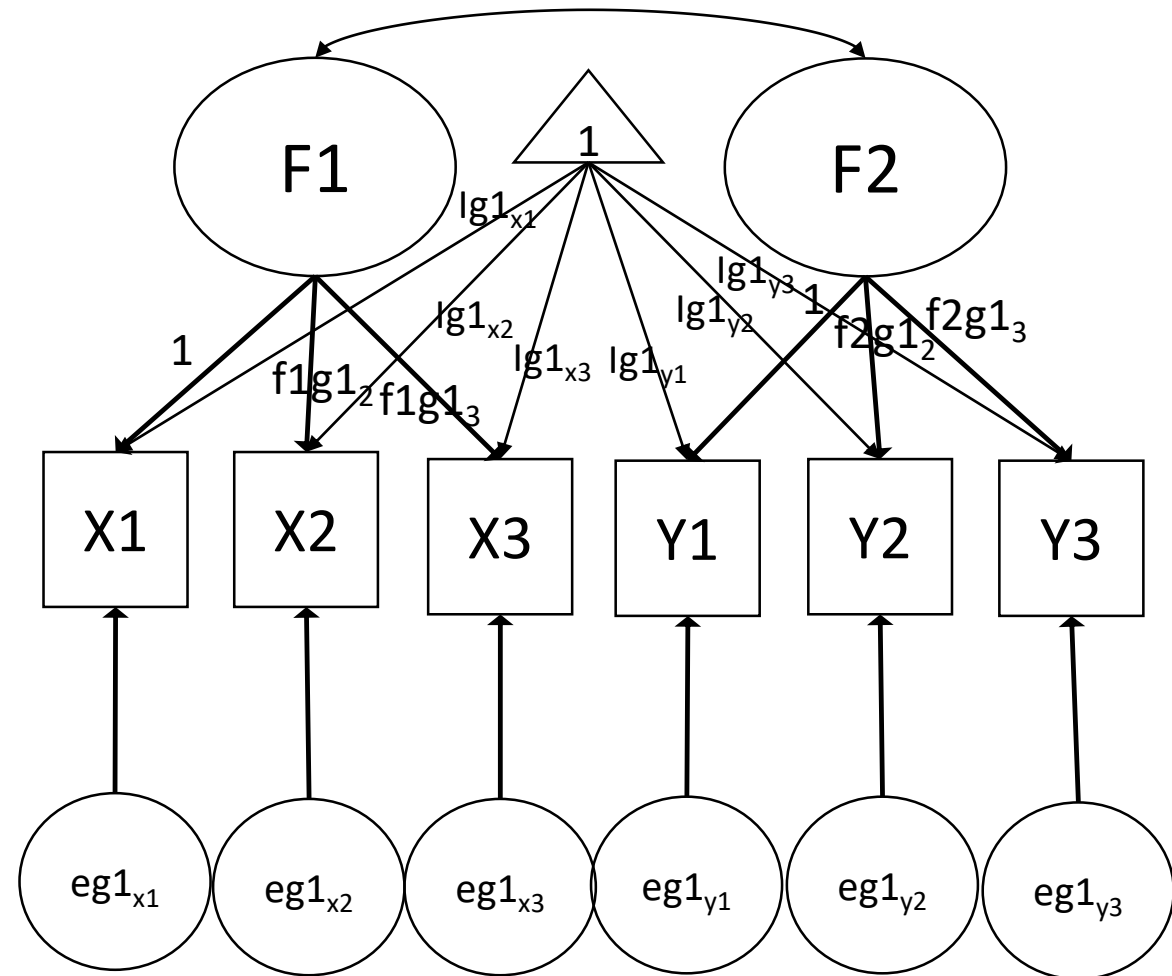


# What about cross-lagged with latent variables?

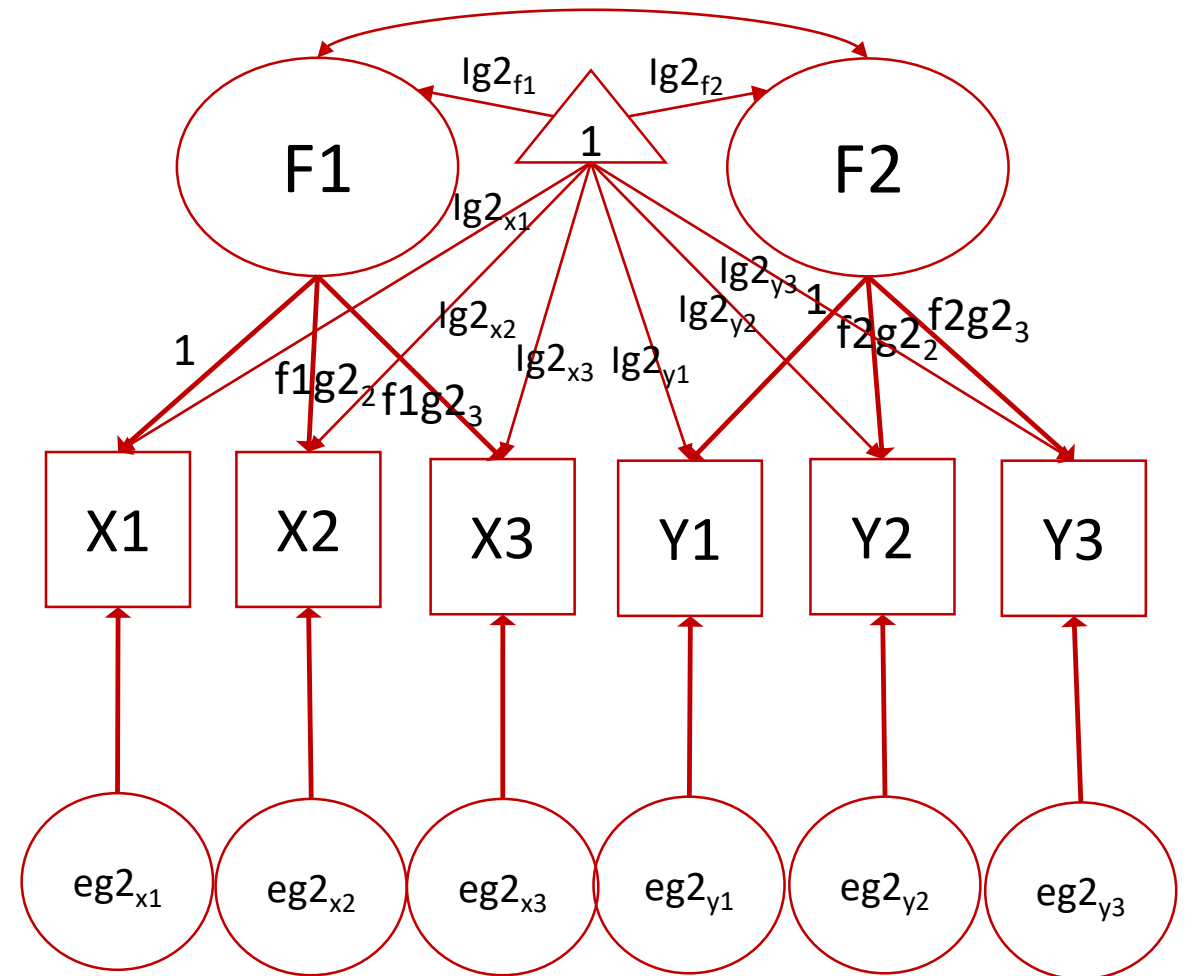
- Before specifying the model, we need to check **measurement invariance**
  - psychometric equivalence of a construct across groups or across time.
  - demonstrates that a construct has the same meaning across groups or repeated measurements.
- Levels of invariance
  1. **Configural** – same pattern of loadings
  2. **Metric (weak factorial)** – equal loadings
  3. **Scalar (strong factorial)** – equal loadings and intercept
  4. **Residual (strict factorial)** – equal loadings, intercepts and item error variances

# Configural invariance – same pattern of loadings

Group 1



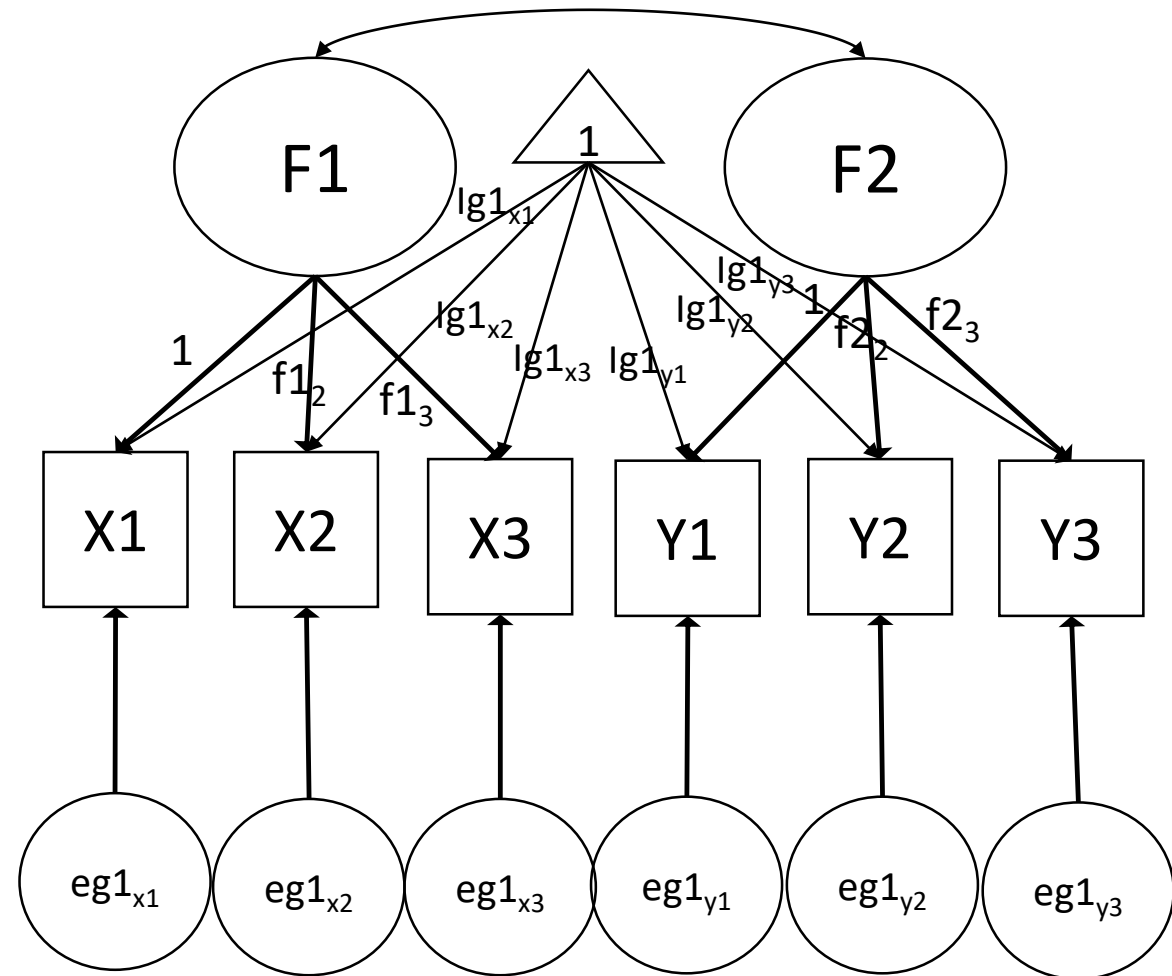
Group 2



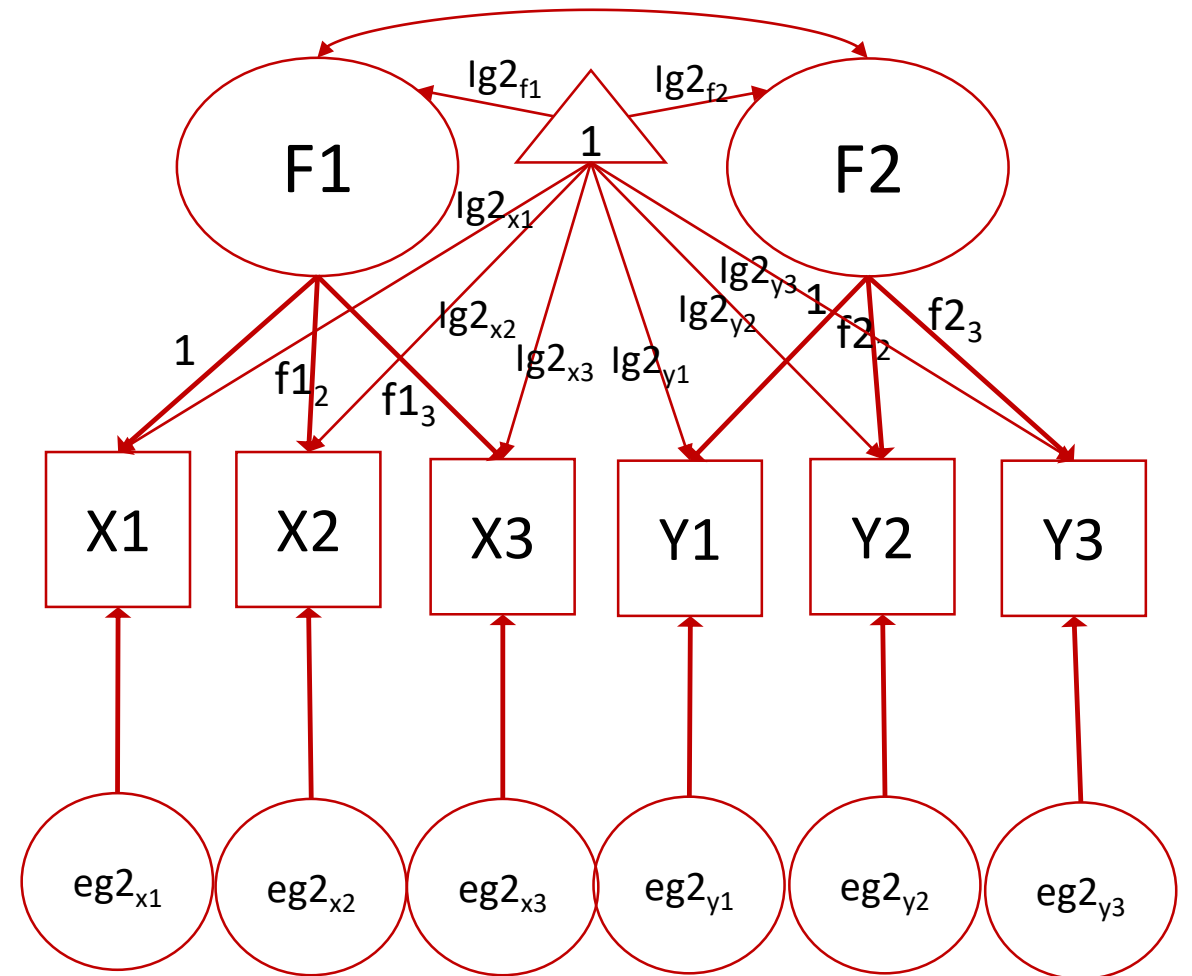


# Metric invariance – equal loadings

Group 1



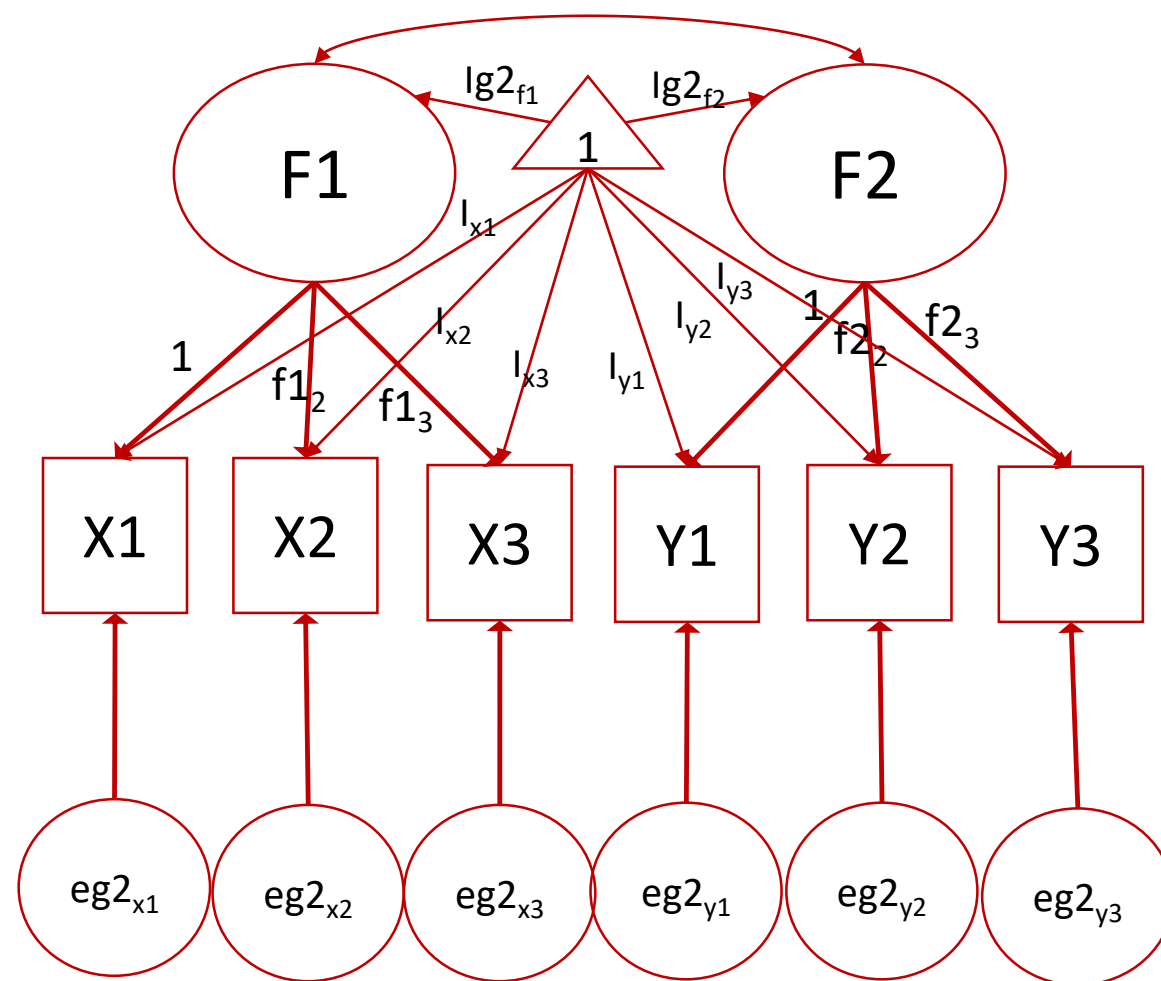
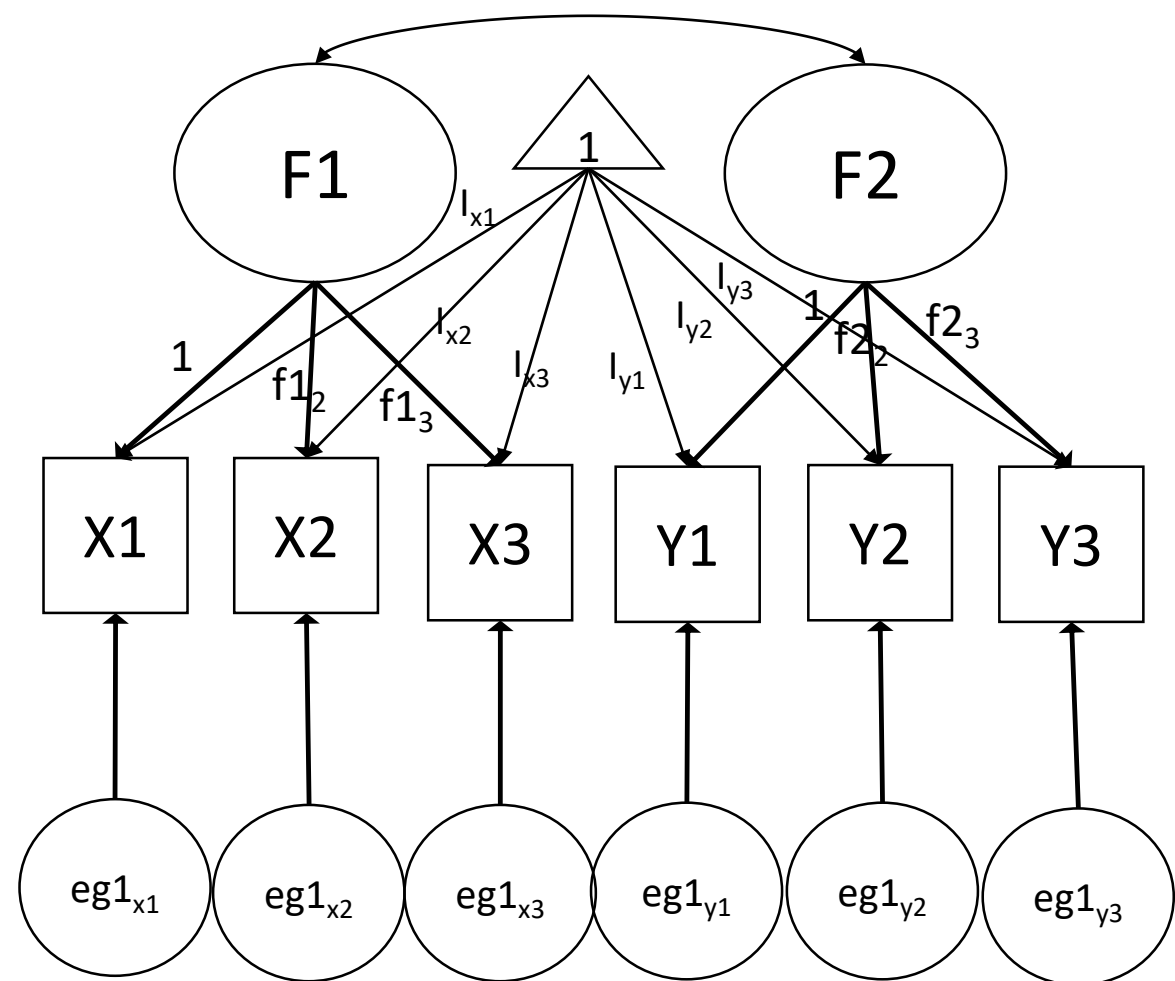
Group 2



# Scalar invariance – equal loadings and intercept (a precondition for comparing latent means across groups)

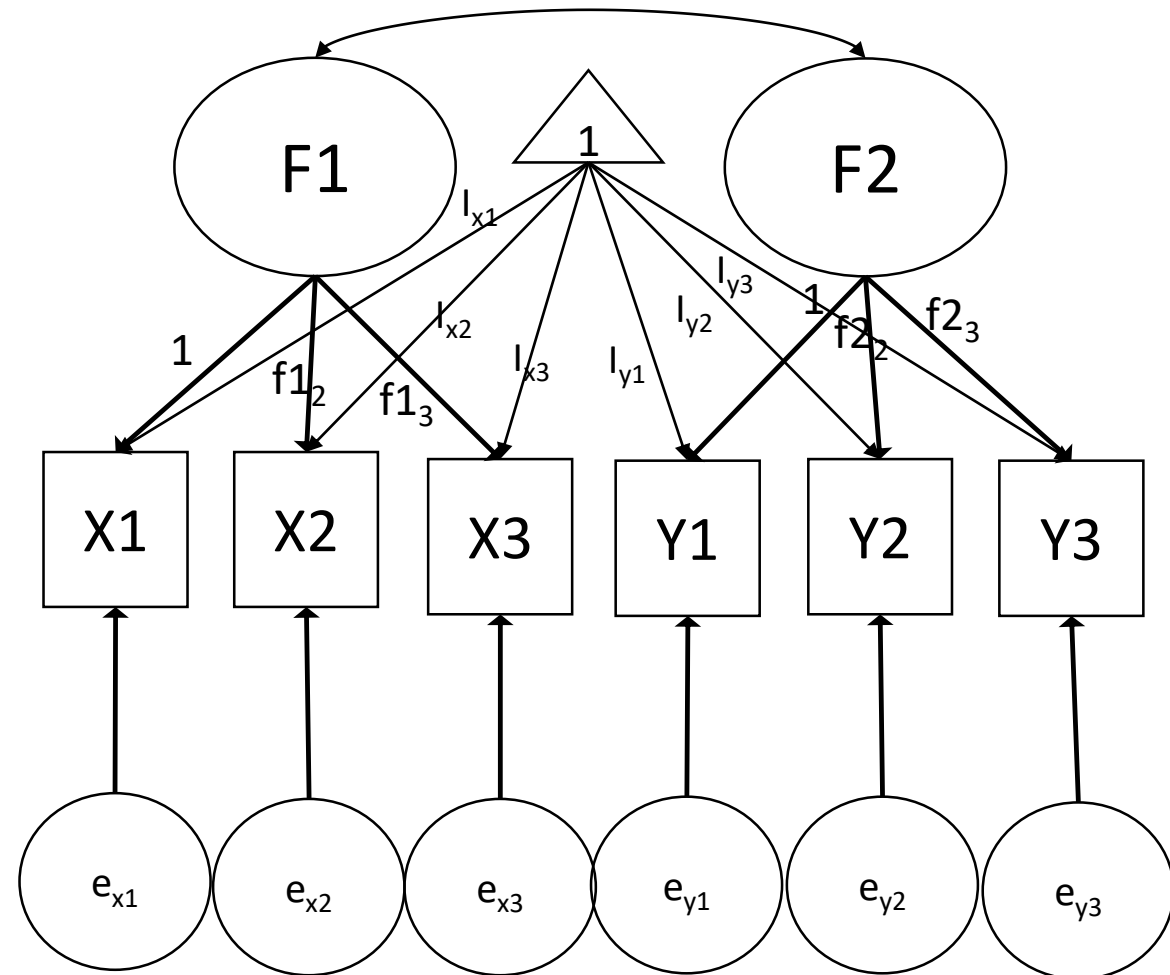
Group 1

Group 2

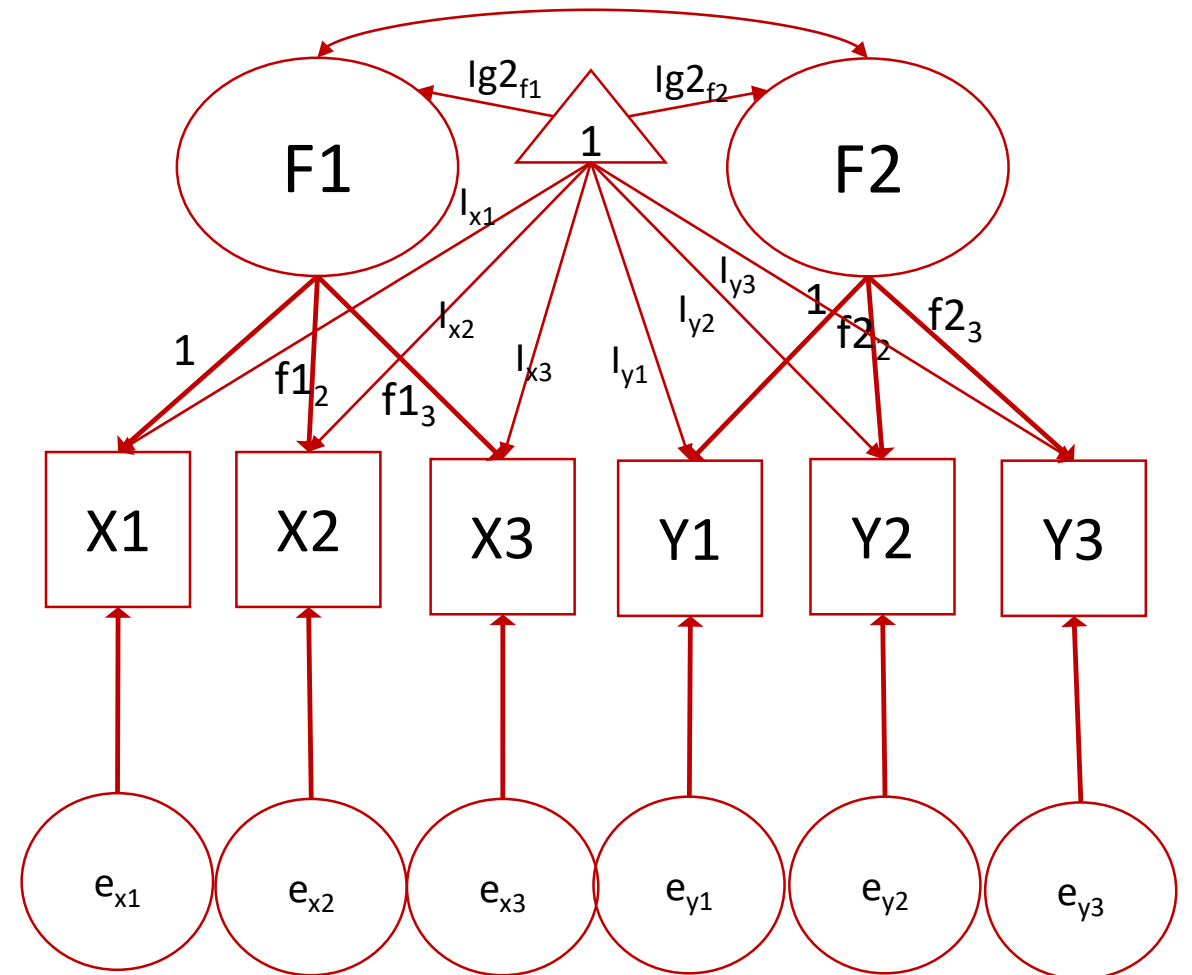


# Residual invariance – equal loadings, intercepts and item errors

Group 1



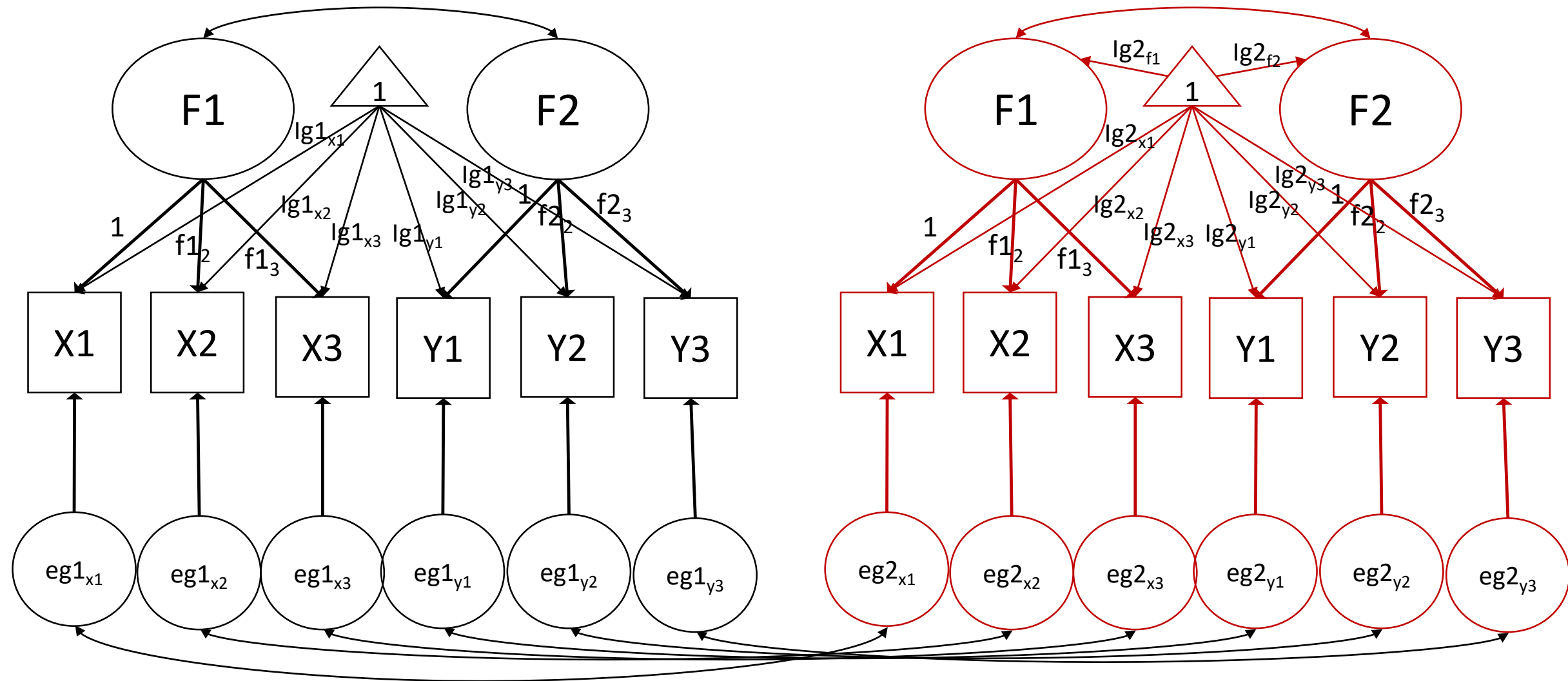
Group 2



# Measurement invariance across time

Time 1

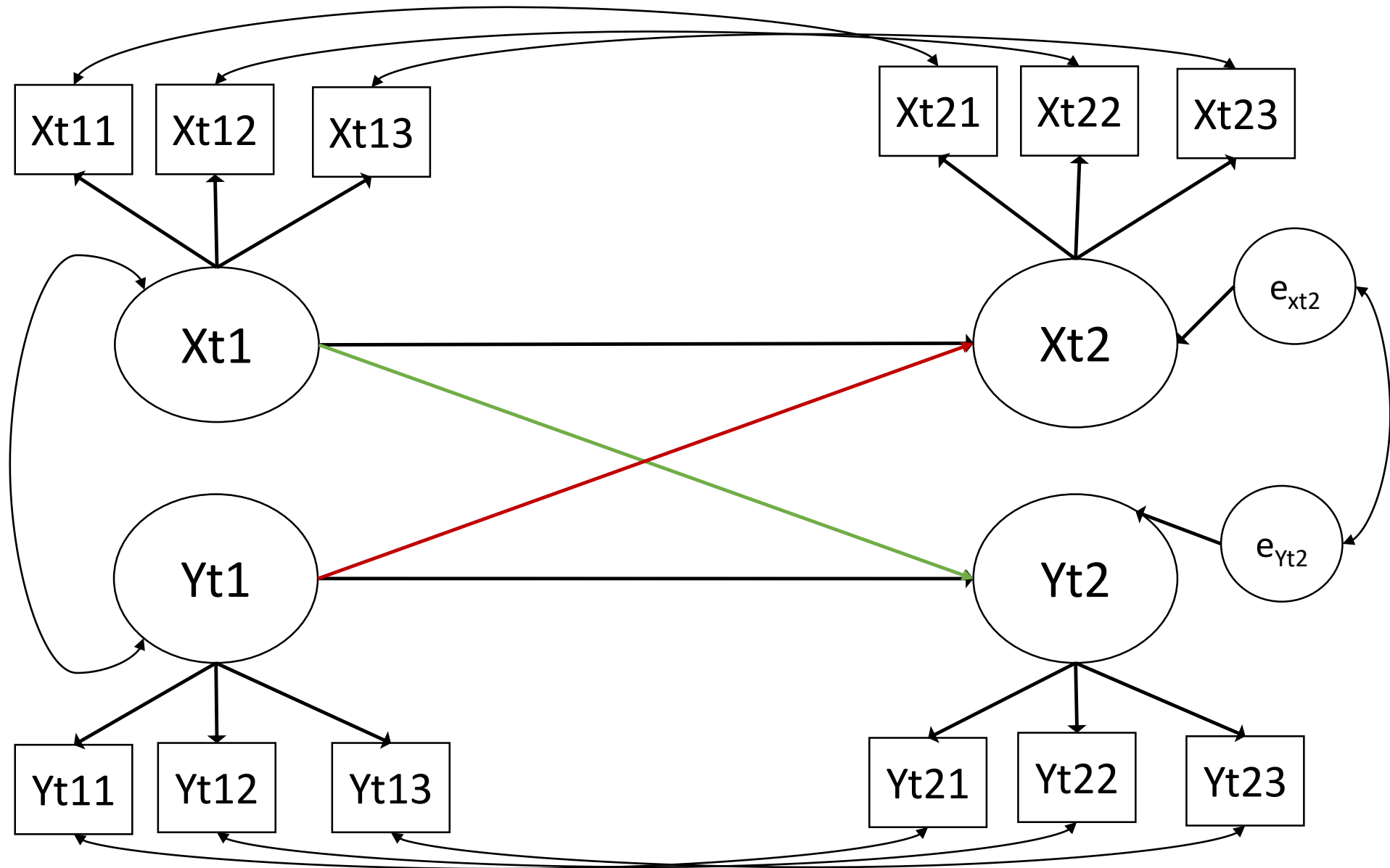
Time 2



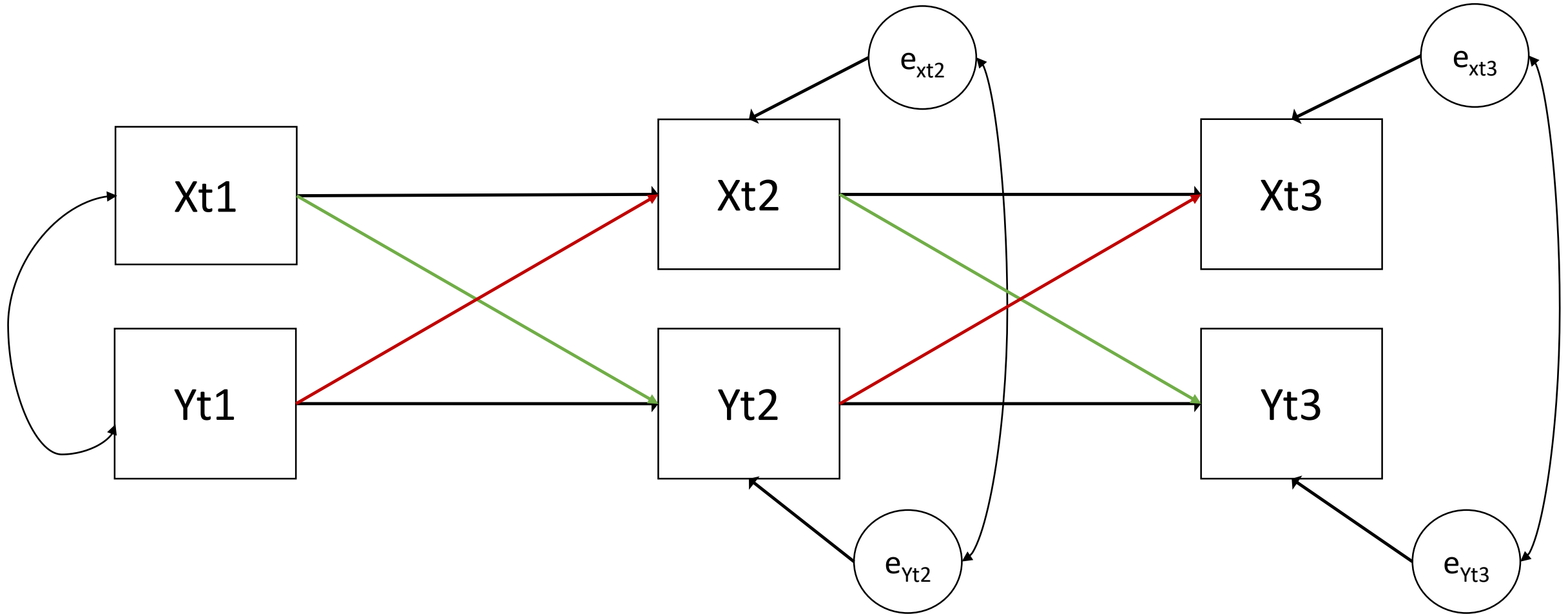
# Measurement invariance across time

- Metric invariance is necessary, scalar is ideal.
- If scalar is not achieved, we can opt for partial scalar invariance – constraining only some (not less than 50%) intercepts to be equal across groups/time points
- Little (2013) suggests that residual invariance is an unrealistic assumption – one should always expect some variation in item indicator errors stemming from random noise/error.

# Cross-lagged model with latent variables



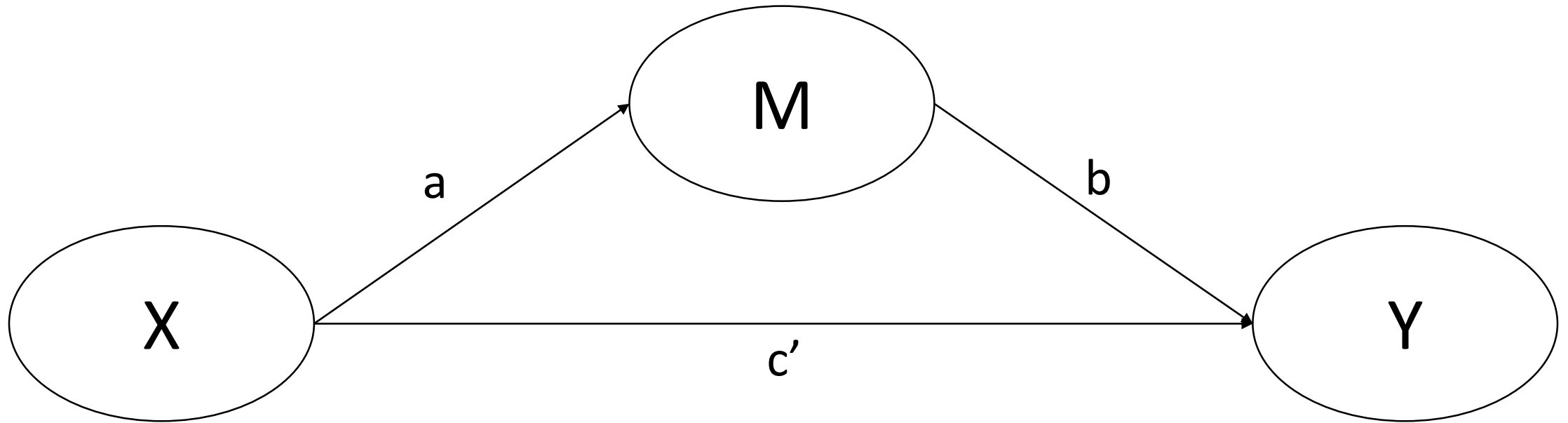
# Cross-lagged model with three time points



# Mediation analysis



# Mediation



- Indirect effect =  $a*b$
- Direct effect =  $c'$
- Total effect ( $c$ ) = Indirect + direct effect ( $a*b + c'$ )

The moderator–**mediator** variable distinction in social psychological research: Conceptual, strategic, and statistical considerations.

RM Baron, DA Kenny - Journal of personality and social ..., 1986 - psycnet.apa.org

In this article, we attempt to distinguish between the properties of moderator and **mediator** variables at a number of levels. First, we seek to make theorists and researchers aware of the ...

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# Mediation analysis

- Baron and Kenny (1986) classic approach to mediation-testing
  1. Check if the relationship between x and y is significant
    - No -> no mediation.
    - Yes -> step 2
  2. Check if the relationship between x and m is significant
    - No -> no mediation.
    - Yes -> step 3
  3. Check if the relationship between m and y is significant
    - No -> no mediation.
    - Yes -> step 3
  4. If we control for m, does the relationship between x and y become non-significant (full mediation) or decreases (partial mediation)

# Mediation analysis

- Baron and Kenny's approach was created when computers were not accessible to everyone.
- In addition, some of its assumptions are too restrictive and probably led to the non-detection of some mediation effects (Lebreton et al., 2008).
- It is much more efficient and accurate to estimate the significance of mediation by simply multiplying the effects of a and b and estimating their significance, i.e., estimating significance of indirect effects.

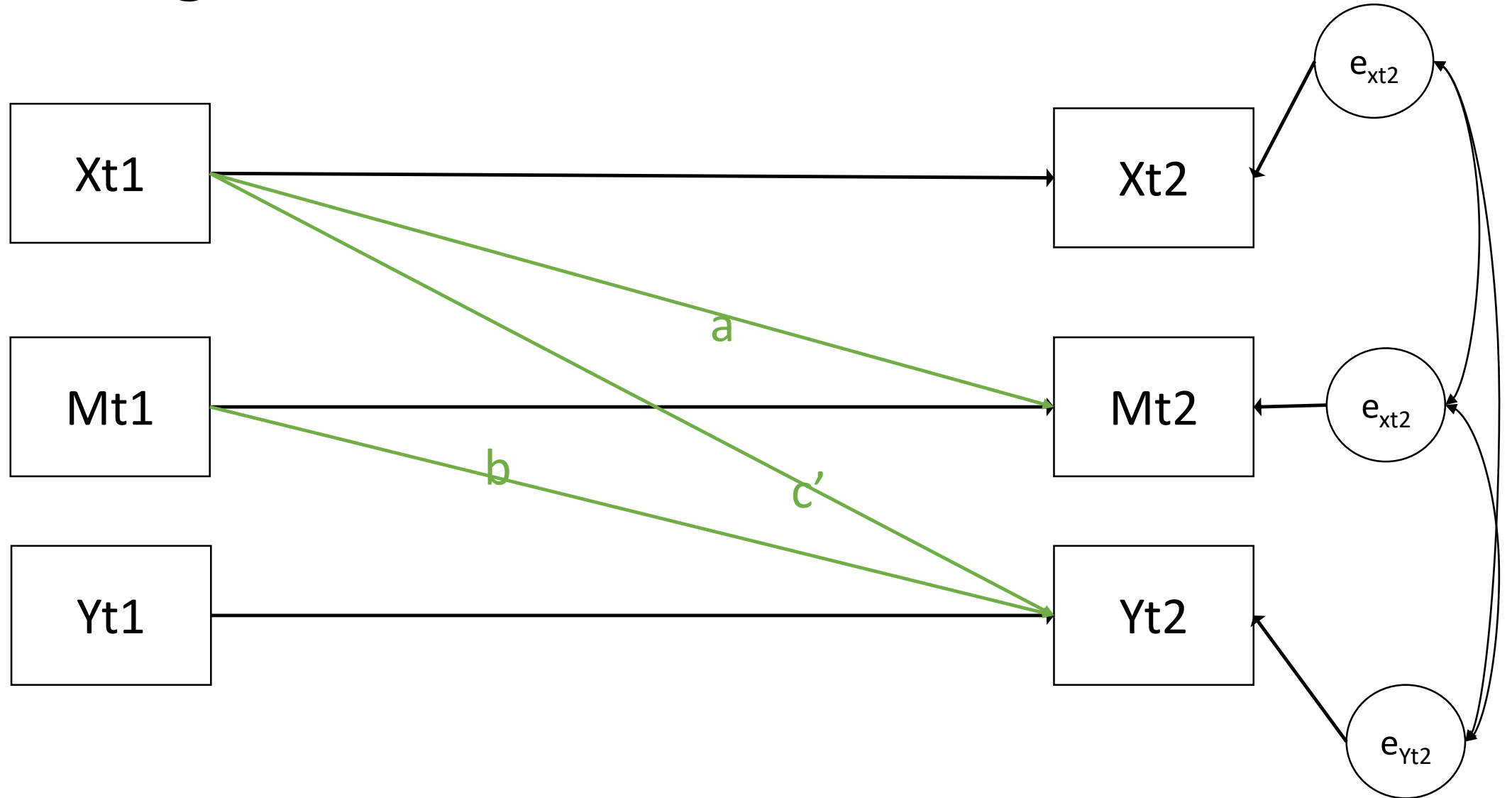
# Indirect effect inference

- The distribution of the  $a*b$  effect is mostly not normal.
- Significance of the indirect effect needs to be assessed using the empirically derived distribution of indirect effects.
- **Bootstrap method** creates a distribution of an indirect effect through resampling process - if the confidence interval obtained using this method **does not include 0**, the indirect effect is **statistically significant**.

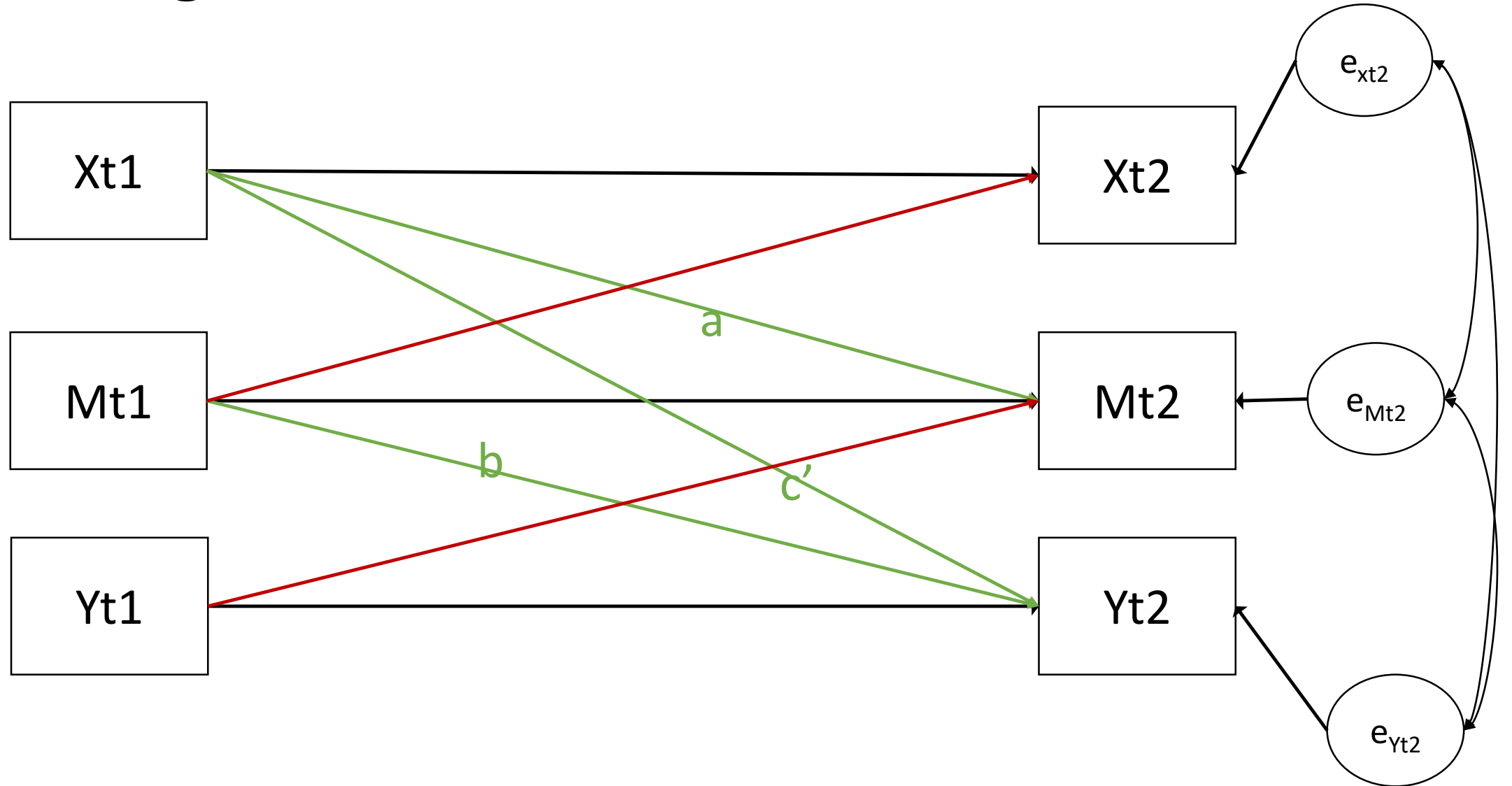
# Mediation analysis

- For mediation in general, a great source is A. F. Hayes and his book Introduction to Mediation, Moderation, and Conditional Process Analysis: A Regression-Based Approach
- It is accompanied by PROCESS macro which is great for mediation analyses without autoregressive effects and cross-lagged relationships.
- <http://afhayes.com/introduction-to-mediation-moderation-and-conditional-process-analysis.html>
- For cross-lagged mediations, we need SEM.

# Half longitudinal model of mediation

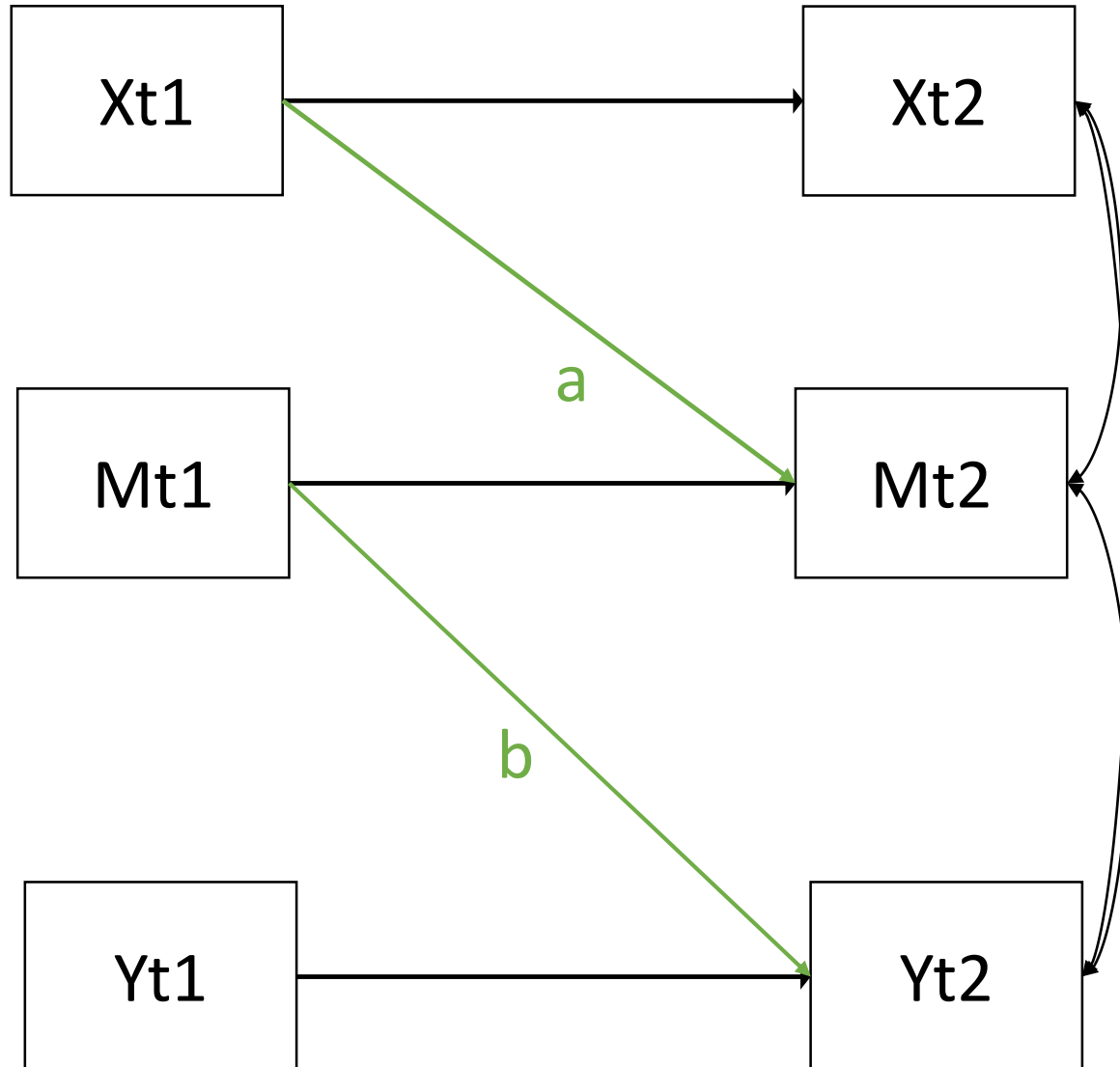


# Half longitudinal model of mediation

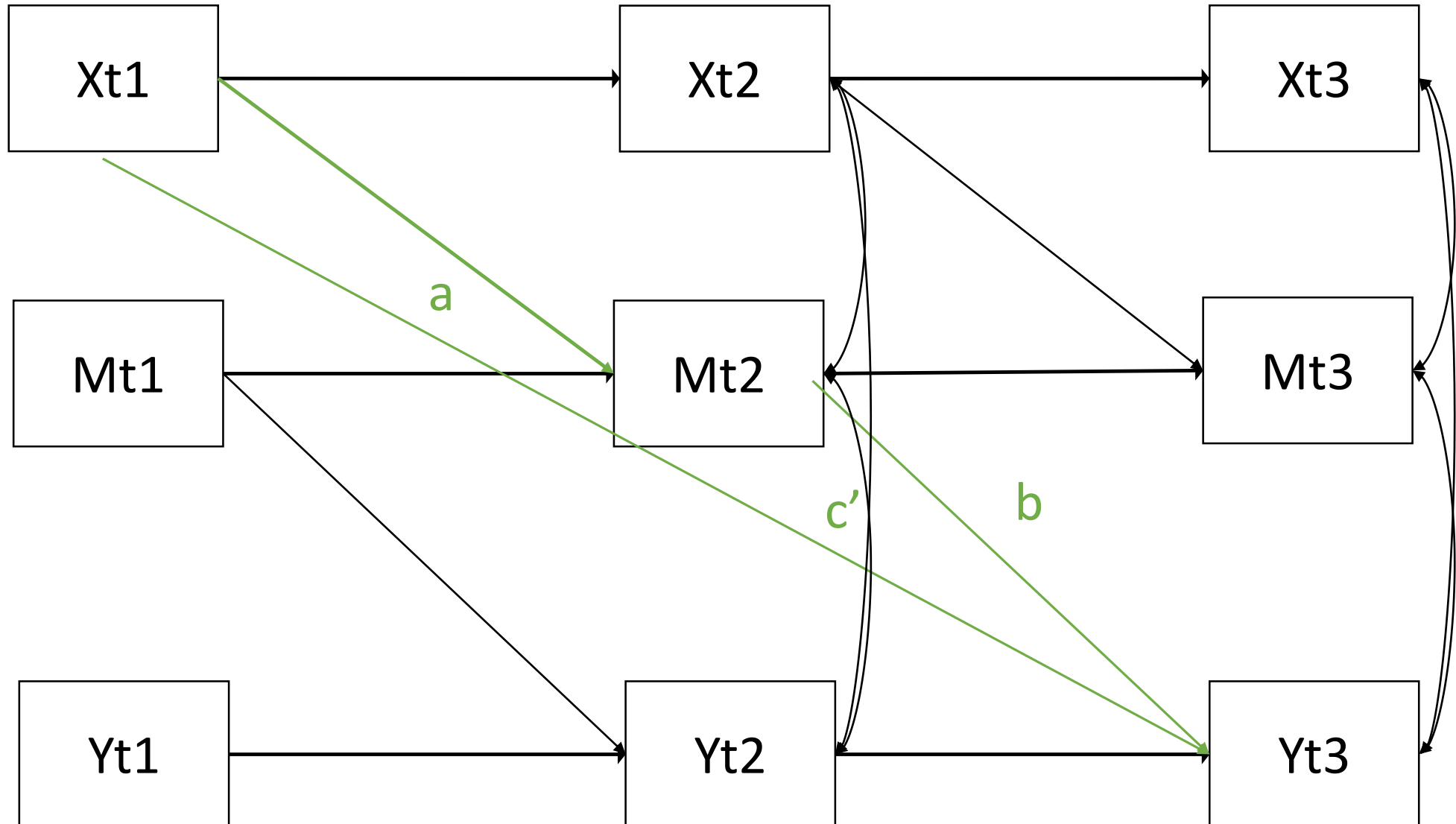




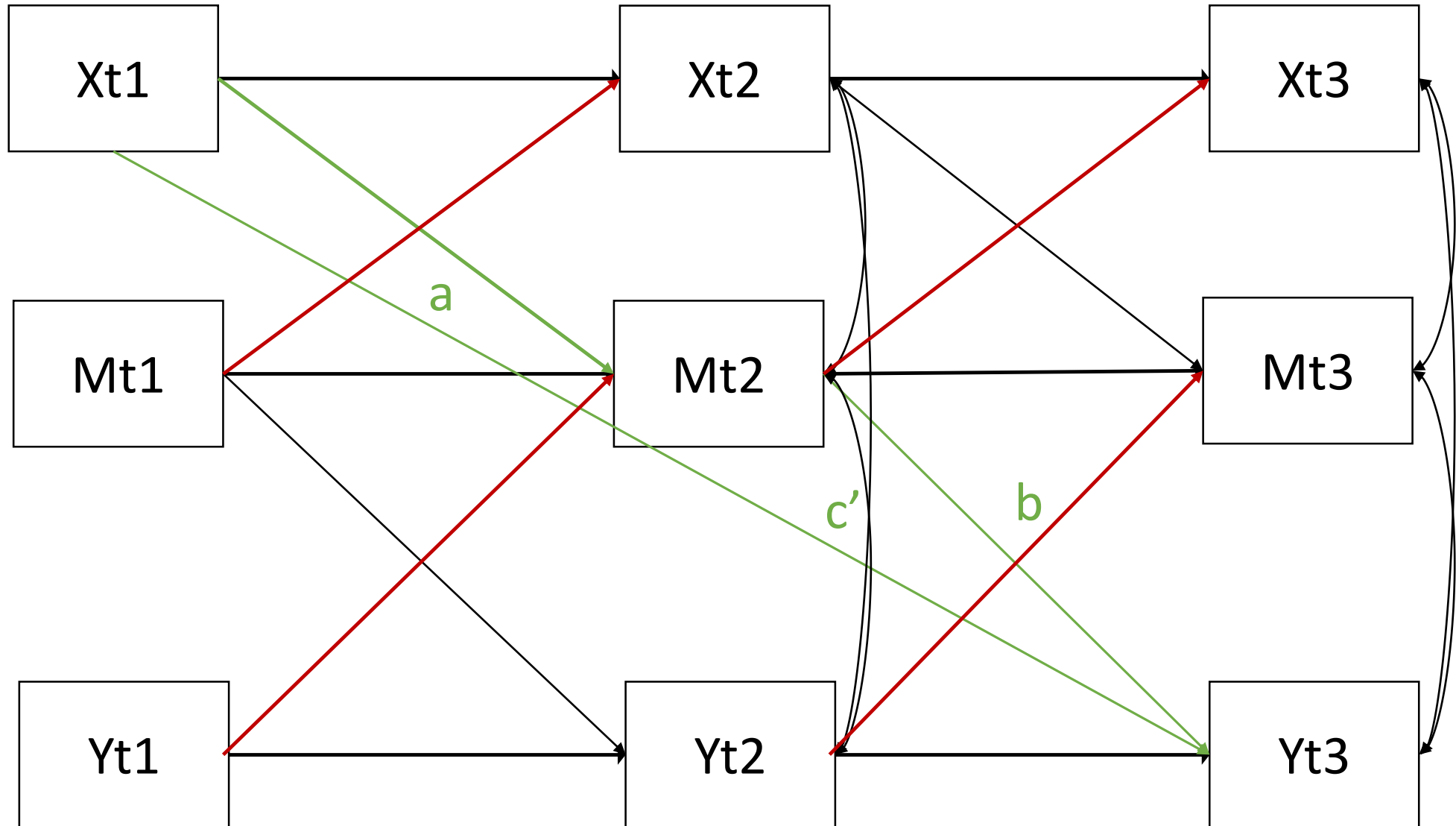
# Full longitudinal mediation



# Full longitudinal mediation



# Full longitudinal mediation



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